Reinforcement Learning in Enormous Action Spaces

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Outline

The large action space problem and discrete action spaces

 Generating Adjacency-Constrained Subgoals in Hierarchical Reinforcement Learning (2020)

Q-Learning in Enormous Action Spaces Via Amortized Approximate Maximization (2020)

Background: Atari and DQNs

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• Atari games have at most 18 actions.

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- Web-scale recommendation systems

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- Training inefficiency in exploration of large action spaces.
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Challenge: scale with action space size $|\mathcal{A}|$

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- Reinforcement Learning in continuous domains often resort to parametric functions for a compact representation of distributions over actions. Often these are Gaussian.
- A discrete policy, in practice, can represent much more flexible distributions than Gaussian when there are sufficient number of atomic actions. Intuitively, a discrete policy can represent a multi-modal action distribution while a Gaussian is by design uni-modal.
- A common argument against a discretized action space is that for an action space with *M* dimensions, discretizing *K* atomic actions per dimension leads to *M^K* combinations of joint atomic actions: *Curse of Dimensionality*.

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- HRL techniques use several forms of abstractions that have the ability to handle the exponentially increasing number of parameters.
- A well-designed reward function in the HRL setting can decrease the number of impractical acts of exploration.

 Goal-conditioned Hierarchical RL comprises of a high-level policy that breaks the original task into a series of subgoals and a low-level policy that aims to reach those subgoals.



Generating Adjacency-Constrained Subgoals in Hierarchical Reinforcement Learning

Tianren Zhang, Shangqi Guo, Tian Tian, Xiaolin Hu, Feng Chen

June 2020

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- The subgoals are interpreted as high-level actions allowing direct training of the meta-controller to generate subgoals using external rewards as supervision.
- **Problem**: Training inefficiency in large goal spaces for the meta-controller. Controller training also suffers as the agent tries to reach every possible subgoal produced by the meta-controller.
- **Proposal**: The high-level action space can be restricted to a k-step adjacent region centered at the current state.

Preliminaries: Notation

- Consider a finite-horizon goal conditioned MDP defined as a tuple (S, G, A, P, R, γ) , where S is a state set, G is a goal set and A is an action set.
- $\mathcal{P}: S \times \mathcal{A} \times S \rightarrow \mathbb{R}$ is the state transition function, $\mathcal{R}: S \times \mathcal{A} \rightarrow \mathbb{R}$ is a reward function, $\gamma \in [0, 1)$ is a discount factor and $\Psi: S \rightarrow G$ is a known mapping function.
- Meta-controller with policy $\pi_{\theta_h}^h(g|s)$, controller with policy $\pi_{\theta_l}^l(a|s,g)$ comprising a two-level hierarchy.

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- Distant subgoals can be substituted by closer subgoals, as long as they drive the controller to move towards the same "direction".
- The meta-controller policy only needs to explore in a subset of subgoals covering states that the controller can possibly reach within k steps.
- For the controller, adjacent subgoals provide a stronger learning signal as the agent can be intrinsically rewarded with a higher frequency for reaching these subgoals.



Figure 1: High-level illustration of our method: distant subgoals g_1 , g_2 , g_3 (blue) can be surrogated by closer subgoals \tilde{g}_1 , \tilde{g}_2 , \tilde{g}_3 (yellow) that fall into the k-step adjacent regions.

Shortest Transition Distance

Definition 1. Let s_1 , $s_2 \in S$. Then, the *shortest transition distance* from s_1 to s_2 is defined as:

$$d_{st}(s_1, s_2) := \min_{\pi \in \Pi} E[T_{s_1 s_2} | \pi] = \min_{\pi \in \Pi} \sum_{t=0}^{\infty} t P(T_{s_1 s_2} = t | \pi)$$

where Π is the complete policy set and $T_{s_1s_2}$ denotes the first hit time from s_1 to s_2 .

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where Π is the complete policy set and $T_{s_1s_2}$ denotes the first hit time from s_1 to s_2 .

An optimal (deterministic) goal-conditioned policy $\pi^*: S \times G \rightarrow A$ is:

$$\pi^*(s,g) \in \operatorname{argmin}_{a \in \mathcal{A}} \sum_{s' \in \mathcal{S}} \mathcal{P}(s'|s,a) d_{st}(s',\Psi^{-1}(g))$$

 $\forall s \in S, \forall g \in G$

k-Step Adjacent Region

Definition 2. Let $s \in S$. Then, the *k*-step adjacent region of *s* is defined as:

$$G_A(s,k) = \{g \in G | d_{st}(s,\Psi^{-1}(g)) \leq k\}$$

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- Consider a goal-conditioned hierarchical policy where the controller is required to reach the subgoals within k limited steps.
- In deterministic MDPs, given an optimal controller policy $\pi^{I*} = \pi^*$, subgoals that fall in the k-step adjacent region of the current state can represent all optimal subgoals in the whole goal space.

Using closer surrogate subgoals

Theorem 1: Let $s \in S$, $g \in G$ and let π^* be an optimal goal-conditioned policy. Under a deterministic MDP with strongly connected states, for all $k \in \mathbb{N}_+$ satisfying $k \leq d_{st}(s, \Psi^{-1}(g))$ there exists a surrogate goal \tilde{g} such that:

 $egin{aligned} & ilde{g}\in \mathit{G}_{\mathsf{A}}(s,k) \ &\pi^*(s_i, ilde{g})=\pi^*(s_i,g) \end{aligned}$

 $\forall s_i \in \tau (i \neq k)$ where $\tau := (s_0, s_1, \cdots, s_k)$ is the k-step trajectory starting from state $s_0 = s$ under π^* and g.

Meta-Controller Optimizing Objective

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- This suggests that the *k*-step action sequence generated by an optimal controller policy conditioned on a distant subgoal can be induced using a subgoal that is closer.
- In the deterministic setting they constrain the meta-controller's action space to state-wise k-step adjacent regions without a loss of optimality.
- Employing relaxation methods they derive the following optimizing objective:

$$\max_{\theta_h} E_{\pi_{\theta_h}} \sum_{t=0}^{T-1} [\gamma^t r_{kt}^h - \eta H(d_{st}(s_{kt}, \Psi^{-1}(g_{kt})), k)],$$

where r_{kt}^h is the reward for the meta-controller's policy, H(x,k) = max(x/k - 1,0) is a hinge loss function, and η is a balancing coefficient. Aggregated Adjacency Matrix and Adjacency Network

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- Use the agent's trajectories to construct and update an aggregate binary k-step adjacency matrix.
- However, the adjacency matrix has a tough time generalizing to newly-visited sets of states and is non-differentiable.



Figure 4: The functionality of the adjacency network. The *k*-step adjacent region is mapped to an ϵ_k -circle in the adjacency space, where $e_{g_i} = \psi_{\theta}(g_i), i = 1, 2, 3.$
Aggregated Adjacency Matrix and Adjacency Network

• Employ an adjacency network, Φ_{ϕ} with parameter ϕ , that learns a mapping from the goal space to an adjacency space ensuring a binary relation for implementing the adjacency constraint.

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• Ex:
$$||\Phi_{\phi}(g_1) - \Phi_{\phi}(g_2)||_2 > \epsilon_k$$
 for $d_{st}(s_1, s_2) > k$ and $||\Phi_{\phi}(g_1) - \Phi_{\phi}(g_2)||_2 < \epsilon_k$ for $d_{st}(s_1, s_2) < k$.

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• The adjacency network is trained by minimizing an objective that penalizes adjacent state embeddings with large Euclidean distances in the adjacency space and non-adjacent state embeddings with small Euclidean distances.

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- With a learned adjacency network, Φ_φ, they incorporate the adjacency constraint into the goal-conditioned HRL framework.
- They minimize the following objective for the meta-controller:

$$\mathcal{L}(\theta_h) = -E_{\pi_{\theta_h}} \sum_{t=0}^{T-1} [\gamma^t r_{kt}^h - \eta \mathcal{L}_{adj}]$$

where $\mathcal{L}_{adj}(\theta_h) \propto \max(||\Phi_{\phi}(\Psi(s_{kt})) - \Phi_{\phi}(g_{kt})||_2 - \epsilon_k, 0)$ is defined as an adjacency loss and $g_{kt} \sim \pi^h_{\theta_h}(g_{kt}|s)$.

Limitations

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- The theorems are derived in the context of the deterministic MDP. They empirically show that their method is robust to **certain** types of stochasticity.

Limitations

- A "k" constraint is manually defined as a hyper-parameter. Thus, extension of this procedure to various settings, that may require a different constraint, is resolved through simulation and cross-validation.
- The theorems are derived in the context of the deterministic MDP. They empirically show that their method is robust to **certain** types of stochasticity.
- For applications with vast state spaces, constructing a complete adjacency matrix will be problematic.

Q-Learning in Enormous Action Spaces Via Amortized Approximate Maximization

Tom Van de Wiele, David Warde-Farley, Adriy Mnih, Volodymyr Mnih

Deepmind. Jan 2020

Q-Learning Flexibility and Complexity

Why doesn't Q-learning scale? Recall Q-learning update:

• Maximization over action space \mathcal{A} .

•
$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha \left(r_t + \gamma \max_{a \in \mathcal{A}} Q(s_{t+1}, a) - Q(s_t, a_t) \right)$$

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- \bullet Computing max on discrete ${\cal A}$ has complexity $\propto |{\cal A}|.$
- Unable to maximize over continuous or hybrid \mathcal{A} .

Extend DQN to Large Action Spaces

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Goal:

- Reduce computational cost dependence on $|\mathcal{A}|$.
- Possibility of continuous/discrete action spaces.

DQN Review

Parameterize state-value function using a neural network Q_{θ} .

Usual case: network maps state input to (action, value) pairs.

- $Q_{\theta} : S \to \mathcal{A} \times \mathcal{V}$
- # parameters $|\theta|$ roughly grows with $|\mathcal{A}|$.



DQN Review: State, Action \rightarrow Value

Can we reduce dependence of network size on action space?

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Can we reduce dependence of network size on action space?

Yes! Redefine Q_{θ} to map (state, actions) to values.

- $Q_{\theta}: \mathcal{S} \times \mathcal{A} \to \mathcal{V}.$
- No explicit size dependence on $|\mathcal{A}|$.
- Forward pass for each action to compute max value.



Amortized Q-Learning (AQL)

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Amortized Q-Learning (AQL)

- Q-learning with cost *less* dependent on $|\mathcal{A}|$.
- Main idea: learn to search for good candidate actions.

• Only estimate value for candidate actions.

• Maximize over (much smaller) proposed set.

AQL: Approach

Learn a proposal distribution μ over possible actions $a \in A$.

- $\mu(a|s; \theta) = \text{probability of being a high-value action.}$
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Parameterize μ with a neural network.

AQL: Q network

- $Q: S \times A \rightarrow V$
- Usual DQN training procedure.



Two NNs: proposal network μ , Q-network Q(s, a).

In a state s:

1 Proposal network forward pass computes $\mu(a \mid s)$.

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2 Sample actions:
$$A_{samp} = A_{\mu} \cup A_{U}$$
, where $A_{\mu} := \{a_i\}_{i=1}^{N}$, $a_i \sim \mu(a \mid s)$
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- **③** Q-network forward pass computes Q(s, a) for each $a \in A_{samp}$.
- Choose optimal action as:

$$a^*(s) = rgmax_{a \in A_{samp}} Q(s, a)$$

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Note: complexity proportional to $|A_{samp}|$, not |A|. (Amortized cost!)

AQL: Proposal network

- Output softmax over actions.
- Train with (regularized) proposal loss:
- $\mathcal{L}(\theta^{\mu}; s) = -\log \mu(a^*(s)|s; \theta^{\mu}) \lambda H(\mu(a|s; \theta^{\mu}))$

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Second term: encourages uncertainty in proposal distribution (exploration).

AQL: Sampling from proposal network

Discrete case:

- Apply softmax to μ output layer.
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Continuous case:

- Option 1: discretize action space.
- Option 2: μ parameterizes a Gaussian:
- I.e. Samples $a_i \sim \mathcal{N}(\mu(a \mid s), \sigma^2)$

AQL: Architecture



AQL: Dependence on $|\mathcal{A}|$

Does AQL address the problem of learning in large action spaces?

AQL: Dependence on $|\mathcal{A}|$

Does AQL address the problem of learning in large action spaces?

- Proposal network has softmax over all actions.
- Why is this better than a DQN predicting (action, value) pairs?
- Both AQL and DQN have an output layer with $|\mathcal{A}|$ nodes.

AQL: Dependence on $|\mathcal{A}|$

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- DQN: learning (action, value) pairs is many regressions.

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- AQL: learning softmax over actions is multi-class classification.
- DQN: learning (action, value) pairs is many regressions.

Intuition: classifying an action as good is "easier" than regressing how good it is.

Results: AQL per-step similar, per-second better than QL



Tom Van de Wiele, David Warde-Farley, Q-Learning in Enormous Action Spaces Via Amortized Approximate Maximization 34 / 37

AQL: Limitations

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Our project: can we exploit structure in action space and reduce complexity dependence on $|\mathcal{A}|$?

Questions?

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